DUAL REALITY

SIEGFRIED GENREITH

ABSTRACT. A major issue for modern physics is how reality in terms of general relativity may emerge from quantum mechanics. Based on my previous proposal for an extension of the QM standard model this paper investigates in a emergent Riemannian space. It shows that this extended model enables a stochastic state reduction process that simply follows geodesic curves in a 4-dimensional spacetime.

Two observations motivate these papers and those that probably will follow:

1) General Relativity with Einstein's Field equation is highly recursive in how it is formulated. The energy distribution determines spacetime geometry and vice versa spacetime geometry determines local trajectories and the evolution of the mass-energy distribution. This feature is absent in quantum mechanics. Here the systems state and observables are different things. A state cannot operate on itself. To introduce recursion in QM requires an extended concept of what the state of a system is.

2) Many papers on collapse or decoherence deal with how observed classical reality is determined by QM. Common understanding is that the Universe evolves according to the Schroedinger equation - i.e. within a unitary U-process. I don't know of any work that considers the Universe - as we observe it - evolving according to a badly understood state reduction process (R-process).

This paper focuses on the second item. It proposes a toy model involving a single spin½ particle. The R-process shows exactly the standard behavior under observations, whereas the internal behavior can be modeled into a 4 dimensional Riemannian space.

INTRODUCTION

This paper and an earlier one had initially been motivated by looking for proceedings in artificial intelligence during the past decades. Though recent models and implementations perform quite successful on complex tasks the gaps are obvious. The understanding of intelligent reasoning and problem solving has not fundamentally improved over the past 20 years. A similar situation can be found in Biology and Psychology. They also fail to explain what really goes on behind intelligence, conscious behavior and consciousness.

Still insufficient but yet most rigorous are attempts to explain intelligent behavior (and finally how consciousness may evolve) by quantum models.

The major objective here is to show that the approach presented here looks promising enough to continue on that path. Its promise is to finally establish one common model that lets QM and GR emerge. And as a side effect we may find very new perspectives into artificial intelligence and conscious behavior.

The proposed extension originally is a discrete integer based model. Extended to reals it is equivalent to a complex model fully isomorphic to QM standard model. The base concept is a triple (A, B, v) of two complex operators A,B and a complex vector v. The operator A is the classical observable, the operator B represents the system, v is a perspective vector so that Bv represents the classical state vector.

On a first sight the model simply adds complexity that seems to be good for nothing. But indeed it provides some additional internal structure. For simple spin½ particles its state B is a complex vector of 8 real dimensions. This structure supports a simple random walk like stochastic process. It is controlled by the external operator A. This process externally behaves exactly as expected from wave reduction: It leaves the state in one of two eigenstates of the observable with the exact probabilities given by the QM standard model. This process experiences external excitations and attractors imprinted by the observable. It works like a grain of sand moving randomly on a sound board and ultimately approaching a wave node. This empirically developed process is quite amazing as it indeed requires 8 real dimensions to work as a simple random walk. Considering classical states with only 4 real dimensions will not produce the correct probabilities. What externally looks like a simple wave reduction caused by measurement now internally leads to concepts of position, movement, acceleration, forces, attractors, time. The entire process can easily be projected to 3 real dimensions. The projected movements within this toy model follow geodesic lines in a curved 4-dimensional spacetime. To achieve the expected results its crucial to leverage both the special GenI specific mapping to the 2-dimensional Hilbert space as well as its discrete structure.

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The first part deals with specifying this stochastic state reduction process and discussing its features to some detail. The second part will focus on a spacetime structure that "absorbs" the process parameters and may be viewed as being imprinted by an external operator. The third part gives some outlook to further investigations in recursion and many particle systems.

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1. Wave Reduction Dynamics

The general hypothesis is a that there is a deep relationship between gravity and wave reduction[4]. The standard view is that wave reduction or decoherence get initiated by gravitational effects performing some sort of a measurement[2]. All those approaches provide partial answers to one or the other question of how reality evolves out of quantum theory. However no comprehensive answer could be provided for many years now. And there is yet no clear vision about a more fundamental model that provides both quantum mechanics (QM) and general relativity (GR) as special cases.

The point of view given here is very different from all these approaches as yet known to me. It suggests that gravity is the major effect describing a systems state reduction process. The objective here is to develop a toy model based on the previous work. The approach takes full advantage of the GenI model – i.e. its 8 real dimensions and its discrete algebraic structure[5]. I'll give a brief overview of that model here.

1.1. The GenI Model

The model frequently referred to extends the QM standard model. Its central concept is an discrete indecomposable algebra A_T with 8 dimensions as a $\mathbb{Z}$ -module. It operates on a 8-dimensional $\mathbb{Z}$ -module A_G that is basically the algebra itself extended to a specific embracing grid. Taken as A_T left modules A_G decomposes into two irreducible left modules.

Its elements get represented as cubes with integers at each node. Such cubes act on each other by performing rotations and reflections combined with sign changes in a special way. There is no fundamental distinction between operators and the states they operate on. Algebraic considerations suggest a mapping to conventional matrices and a concept of states. For convenience the model may be extended to real complex numbers. In this model each system state has a operational feature and may operate on itself.

An observation in this model is after all represented by a triple *(A, B, v)* , with $A, B \in Mat_{\mathbb{C}}(2\text{x}2)$, $v \in \mathbb{C}^2$. Here *A* is the observable, *B* represents the System, v a perspective such that *Bv* is a classical state vector With these conventions it completely resembles the QM standard model of measurements.

The model provides a mapping from the 8 cube nodes to complex matrices by $\begin{pmatrix} b_0 \\ \vdots \\ b_7 \end{pmatrix} \rightarrow \frac{1+i}{2} \begin{pmatrix} b_5 - i\,b_6 & b_4 - i\,b_7 \\ b_1 - i\,b_2 & b_0 - i\,b_3 \end{pmatrix}$. It will be clear from the respective context if *B* is considered as a vector in $\mathbb{R}^8$ or as the corresponding complex operator on $\mathbb{C}^2$. The perspectives are actually restricted to $v = (1+i)\begin{pmatrix} 1 \\ \beta \end{pmatrix};\ \beta \in \{\pm 1 ; \pm i\}$. Whenever I have to map complex vectors to real vectors I'll use the $\mathbb{R}$ -linear mapping $\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \rightarrow \begin{pmatrix} \Re(v_1) \\ \Im(v_1) \\ \Re(v_2) \\ \Im(v_2) \end{pmatrix}$ as a convention.

Because in the following sections I vary node values *(b_0, ... , b_7)* of those cubes let me give some further remarks. If you consider a matrix it looks weird to have a process that changes its elements and then probably reuse it as an operator. On first sight that seems a nice playground that is certainly of no relevance to real world dynamics as it obviously depends on the choice of coordinate systems. But this is not a valid objection here. The GenI model is a purely algebraic construct. There is actually a exposed $\mathbb{Z}$ -base $(p_0, \dots p_7)$ for A_T so that $P \stackrel{\text{def}}{=} \{p_0, \dots, p_7, -p_0, \dots, -p_7\}$ behave isomorphic to the Pauli group. These can be taken as basic entities and each element *A* in A_T can be written simply as a (non unique) series $(A_1, \dots, A_N, 0, \dots)$, $A_i \in P$, $A = \sum A_i$ of these group members. So changing parameters simply may be viewed as a result of creation and deletion processes within this series. It will turn out that the dynamics to be determined are consistent with that view.

1.2. Requirements

Let us look at a given observation (A, B, v) with an hermitian observable A , the system *B*,

$A, B \in Mat_{\mathbb{C}}(2x2)$ and the perspective $v \in \mathbb{C}^2$. This observation may represent the spin measurement of a spin½ particle. The objective now is to design a stochastic process $(B_n(\omega))$ [1] within the 8 dimensional real space representing possible instances of *B*.

The hard requirements are

- This process must completely be controlled by A and v.
- It starts with $B_0 = B$ and must end up with the discrete distribution $B_n v \underset{n\to\infty}{\to} B_\infty v;\ P(B_\infty v = a_1) = |\alpha|^2\ \ P(B_\infty v = a_2) = 1 - |\alpha|^2$. Here $a_i : |a_i| = 1$ are eigenvectors of *A*, $Bv = \gamma(\alpha a_1 + \beta a_2)$, $|\alpha|^2 + |\beta|^2 = 1$. The limiting distribution must be guaranteed at least for the standard observables for spin measurements represented by the Pauli matrices with eigenvalues ± 1.
- It must not depend on the scale of *B*: *B* and αB as a starting point must lead to the same process behavior.

Further requirements are intuitively reasonable

- The process should be locally controlled by *A* without previous knowledge of eigenvectors. The process dynamics should even work without existing real eigenvectors at all.
- The process should be simple in the sense, that it does not require fine tuning of any parameters and involves rather simple probability distributions.

1.3. Process Design

I will give here one process that proves to match the requirements for state reduction of a single spin ½ particle. The process has been developed empirically involving simulations and comprehensive statistical analysis[2].

The base ingredients are the concepts of the discrete GenI model mapped to operators and vectors of a 2-dimensional complex Hilbert space. These concepts include the semi scalar product given by $[\![A|B]\!] \overset{\text{def}}{=} \Re\langle Av|Bv\rangle$ - i.e. the real part of the standard Hilbert product - , the semi norm $[\![A]\!]^2 \overset{\text{def}}{=} [\![A|A]\!] = |Av|^2$ and the angle $\Phi_A(B) \in [0; \frac{\pi}{2}]$ given by $\cos(\Phi_A(B)) = \frac{[\![AB|B]\!]}{[\![AB]\!][\![B]\!]};\ [\![AB]\!], [\![B]\!] > 0$ and $\Phi_A(B) = \frac{\pi}{2}$ else. Φ_A thus defines a scalar field on the state space $S_B \overset{\text{def}}{=} \{B \in \mathbb{R}^8 : [\![B]\!] > 0, [\![AB]\!] > 0\}$ of *B*.

I have to take *B* as a vector $(b_0, \ldots, b_7)$ corresponding to the GenI representation $B = \frac{1+i}{2}\begin{pmatrix} b_5 - i b_6 & b_4 - i b_7 \\ b_1 - i b_2 & b_0 - i b_3 \end{pmatrix}$. Within this vector space I choose $\langle .|.\rangle$ and $|.|$ as the standard euclidean scalar product and norm.

The process is now defined as a two step procedure. Given B_n I have to determine $B_{n+1}(\omega) = D(\omega, B_n) + B_n$ and let $D(\omega, B) = D_1(\omega, D_0(\omega, B), B)$ [3].

1. I start with a random walk step $\Delta B = D_0(\omega, B_n)$ in any direction limited by $|\Delta B| \le \sin(\Phi_A(B))^2 [\![B_n]\!]$. The excitation $\sin(\Phi_A)^2$ is *=0* if *Bv* is an eigenvector of *A* and *=1* if $Bv \perp ABv$. So any movement gets extremely slow near and stops at an eigenvector.
2. The second step $D_1(\omega, \Delta B, B_n)$ randomizes on the decision if to modify B_n by ΔB or $-\Delta B$.

To parameterize the first step $\Delta B = D_0(\omega, B)$ I select independent $\Delta b_i \in [-1; 1];\ d \in [0; \sin(\Phi_A(B))^2 [\![B_n]\!]]$ uniformly distributed and define $\Delta B \overset{\text{def}}{=} \frac{d}{\sqrt{\sum (\Delta b_i)^2}}(\Delta b_0; \ldots; \Delta b_7)$.

The second step now introduces a gradient of the scalar field Φ_A to the process. Given B_n and ΔB the conditional distribution of B_{n+1} is given by

1 The probability measure may be taken as the respective Lebesgue measure on $[0;1]^m$.
2 Applet "GenI Model" can be found on http://bzus.de/literatur.html

$$P(B_{n+1}(\omega)=B_n \pm \Delta B)=P(D_1(\omega, \Delta B, B_n)=\pm \Delta B)=\frac{\Phi_A(B_n \mp \Delta B)}{\Phi_A(B_n+\Delta B)+\Phi_A(B_n-\Delta B)} .$$

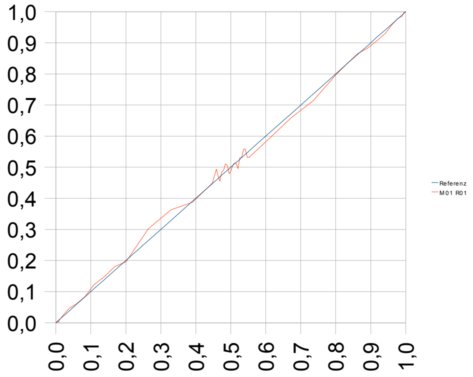

FIGURE 1.CONFORMAL R-PROCESS

Simulations show, that indeed this process provides the expected results globally for any starting point B_0 and Pauli matrices *A*. The graphic shows expected probabilities versus simulated statistics. An eigenvector of *A* is reached with a probability according to $|\lambda_i|^2$ in $B=\lambda_0 b_0+\lambda_1 b_1$. The graphic shows the actual probability values as the final result of the process versus the theoretical values $\frac{x^2}{x^2+1}$ for $x=\frac{|\lambda_0|}{|\lambda_1|}\in[0;1]$ .The sample size was 1000 calculations per selected amplitude of $B_0 v$ running each 250 iterations within the process.

1.3.1. EXCURSUS

To calculate some integrals later on I parameterize in polar coordinates. All elements on the surface of the unit circle receive equal probabilities. Let $K_n=\begin{pmatrix} x_n \\ \vdots \\ x_1 \end{pmatrix}=r\begin{pmatrix} \cos(\phi_{n-1}) \\ \sin(\phi_{n-1})\cos(\phi_{n-2}) \\ \vdots \\ \sin(\phi_{n-1})\ldots\sin(\phi_2)\cos(\phi_1) \\ \sin(\phi_{n-1})\ldots\sin(\phi_2)\sin(\phi_1) \end{pmatrix}$ and the according functional matrix

$$det\frac{\partial(x_n,\ldots,x_1)}{\partial(r,\phi_{n-1},\ldots,\phi_1)}=r^{n-1}det\begin{pmatrix} \cos(\phi_{n-1}) & -\sin(\phi_{n-1}) & 0 & \cdots & 0 \\ \sin(\phi_{n-1})\cos(\phi_{n-2}) & \cos(\phi_{n-1})\cos(\phi_{n-2}) & -\sin(\phi_{n-1})\sin(\phi_{n-2}) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sin(\phi_{n-1})\ldots\sin(\phi_2)\cos(\phi_1) & \cos(\phi_{n-1})\ldots\sin(\phi_2)\cos(\phi_1) & \sin(\phi_{n-1})\ldots\sin(\phi_2)\cos(\phi_1) & \cdots & -\sin(\phi_{n-1})\ldots\sin(\phi_2)\sin(\phi_1) \\ \sin(\phi_{n-1})\ldots\sin(\phi_2)\sin(\phi_1) & \cos(\phi_{n-1})\ldots\sin(\phi_2)\sin(\phi_1) & \sin(\phi_{n-1})\ldots\sin(\phi_2)\sin(\phi_1) & \cdots & \sin(\phi_{n-1})\ldots\sin(\phi_2)\cos(\phi_1) \end{pmatrix} .$$

A volume element is now $dV_n=det\frac{\partial(x_n,\ldots,x_1)}{\partial(r,\phi_{n-1},\ldots,\phi_1)}dr\, d\phi_{n-1}\cdots d\phi_1$ and the surface element $dO_n=\frac{dV_n}{dr}=det\frac{\partial(x_n,\ldots,x_1)}{\partial(r,\phi_{n-1},\ldots,\phi_1)}d\phi_{n-1}\cdots d\phi_1$. Thus I get a probability density $D_n(\phi_1,\ldots,\phi_{n-1})=\frac{1}{O_n}det\frac{\partial(x_n,\ldots,x_1)}{\partial(r,\phi_{n-1},\ldots,\phi_1)}|_{r=1}$.

If I develop along the 1st line I get

$$D_n(\phi_1,\ldots,\phi_{n-1})=\frac{1}{O_n}\left[\frac{\partial x_n}{\partial r}det\frac{\partial(x_{n-1},\ldots,x_1)}{\partial(\phi_{n-1},\ldots,\phi_1)}|_{r=1}-\frac{\partial x_n}{\partial\phi_{n-1}}\frac{\partial(x_{n-1},\ldots,x_1)}{\partial(r,\phi_{n-2},\ldots,\phi_1)}|_{r=1}\right] =$$

$$\frac{1}{O_n}\left[\cos(\phi_{n-1})det\frac{\partial(x_{n-1},\ldots,x_1)}{\partial(\phi_{n-1},\ldots,\phi_1)}|_{r=1}+\sin(\phi_{n-1})\frac{\partial(x_{n-1},\ldots,x_1)}{\partial(r,\phi_{n-2},\ldots,\phi_1)}|_{r=1}\right] =$$

$$\frac{O_{n-1}}{O_n}D_{n-1}(\phi_1,\ldots,\phi_{n-2})\left[\cos(\phi_{n-1})^2\sin(\phi_{n-1})^{n-2}+\sin(\phi_{n-1})^n\right] = \frac{O_{n-1}}{O_n}D_{n-1}(\phi_1,\ldots,\phi_{n-2})\sin(\phi_{n-1})^{n-2} .$$

Indeed the ϕ_i are independent stochastic variables with probability densities $d_i(\phi_i)=\frac{1}{C_{i-1}}\sin(\phi_i)^{i-1};\ d_1=\frac{1}{2\pi}$

where $C_n \overset{\text{def}}{=} \int_0^{\pi} \sin(\Phi)^n d\Phi = \frac{n-1}{n} C_{n-2};\ C_0=\pi;\ C_1=2$.

Some values are $C_2=\frac{\pi}{2};\ C_3=\frac{4}{3};\ C_4=\frac{3}{8}\pi;\ C_5=\frac{16}{15};\ C_6=\frac{15}{48}\pi;\ C_7=\frac{32}{35};\ C_8=\frac{35}{128}\pi$ and according densities $d_2=\frac{1}{2}\sin(\Phi_2);\ d_3=\frac{2}{\pi}\sin(\Phi_3)^2;\ d_4=\frac{3}{4}\sin(\Phi_4)^3;\ d_5=\frac{8}{3\pi}\sin(\Phi_5)^4;\ d_6=\frac{15}{16}\sin(\Phi_6)^5;\ d_7=\frac{16}{5\pi}\sin(\Phi_7)^6$.

For n=8 this means for the density $D_8(\Phi_1,\dots,\Phi_7)=\frac{16}{5\pi} D_7(\Phi_1,\dots,\Phi_6)\sin(\Phi_7)^6$.

The following integral represents the expectation value of $\cos(\Phi_7)^2$ and resolves to $\int_0^{\pi} \cos(\Phi_7)^2 D_8(\Phi_1,\dots,\Phi_7)\, d\Phi_1 \dots d\Phi_7 = \frac{16}{5\pi} \int \cos(\Phi_7)^2 \sin(\Phi_7)^6 d\Phi_7 \int D_7(\Phi_1,\dots,\Phi_6) d\Phi_1 \dots d\Phi_6$ $= \frac{16}{5\pi} \int \left(\sin(\Phi_7)^6 - \sin(\Phi_7)^8\right) d\Phi_7 = 1 - \frac{16}{5\pi} \int_0^{\pi} \sin(\Phi_7)^8 d\Phi_7 = 1 - \frac{16}{5\pi}\frac{35\pi}{128} = \frac{1}{8}$.

1.4. Review on Alternative Process Designs

Empirically approaching the final process above I started with defining it as a optimization problem for the target function $t_A(B)=\frac{[\![AB|B]\!]}{[\![AB]\!][\![A]\!]}$. This target has extreme values ± 1 exactly where *Bv* is an eigenvector of *A*. This definition also matches the base requirements of scale invariance.

First approaches involved a vector field on S_B where at each step the direction *B* to *AB* got fed back to the process with different scalings and randomizations. All those extensive attempts could not successfully simulate a state reduction process. The final distribution always ended up with roughly a fifth order dependency on amplitudes of *B*. For any normalized $B=\alpha a_1+\beta a_2$ I got probabilities for $B=a_1$ as about $\frac{\alpha^5}{\alpha^5+\beta^5}$ instead $\frac{\alpha^2}{\alpha^2+\beta^2}$.

First more promising designs involved a random walk not feeding back any direction. However some feedback obviously is needed. Now leaving up the line of change completely randomly and only feeding back a scalar indicating a preferred direction of change along that freely chosen line finally succeeded. This drilled down the order to the correct order 2. However first attempts provided bad convergence and systematically wrong statistics especially for small amplitudes.

So the definitions for both steps of the actual process are critical to this conformal behavior. If I take the excitation to be Φ^2 or something like $1-\cos(\Phi)$ or $\sin(\Phi)$ instead of $\sin(\Phi)^2$ the process will fail to produce correct limit distributions. The same is true if I use something else than Φ to define the scalar field in the second step.

Quite interesting is that varying simply the real and imaginary parts of the complex matrix B according to the standard Hilbert product on $\mathbb{C}^4$ also leads to the wrong statistics. The second graphic shows the results of one such simulation. To get it working I have to decrease the process excitation by a factor $\sqrt{2}$ resulting from the transformation of the GenI model to the standard model.

So it seems that the GenI model is fundamental for this sort of dynamics.

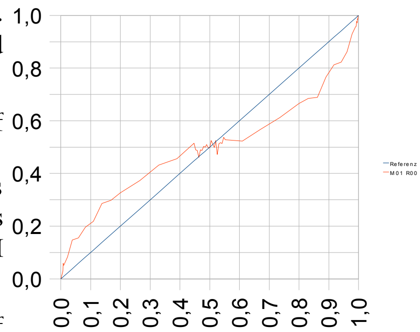

Figure 2: process statistics using Hilbert product

1.5. Process Features

The standard process defined so far as a source for state reduction has been empirically developed considering basic symmetry requirements. It is not clear at all to me how to characterize a entire class of such processes as a pool of candidates. So I will focus on the features of this single candidate.

Because $P\big(D_1(\omega,\alpha\Delta B,\alpha B)=\alpha\Delta B\big)=P\big(D_1(\omega,\Delta B,B)=\Delta B\big)=P\big(\alpha D_1(\omega,\Delta B,B)=\alpha\Delta B\big)$ the stochastic functions $D_1(\omega,\alpha\Delta B,\alpha B)\sim\alpha D_1(\omega,\Delta B,B)$ are equally distributed. Furthermore is $D_0(\omega,\alpha B)\sim|\alpha|D_0(\omega,B)$ and hence for the entire process $\alpha>0\Rightarrow D(\omega,\alpha B)\sim\alpha D(\omega,B)$.

The expectation value for ΔB is always zero $E(\Delta B)=0$.

For small angles $\Phi_A(B)\ll\frac{\pi}{2}$ the excitation $dB=(db_j)$ is small with $|dB|\leq\sin(\Phi_A(B))^2[\![B]\!]\ll[\![B]\!]$.

So near an eigenvector of A the approximate conditional distribution is given by $P(D_1(\omega,dB,B)=\pm dB)=\frac{\Phi_A(B\mp dB)}{\Phi_A(B+dB)+\Phi_A(B-dB)}\sim\frac{\phi_A(B)\mp\langle\nabla\phi_A|dB\rangle}{2\phi_A(B)}=\frac{1}{2}\mp\frac{\langle\nabla\phi_A|dB\rangle}{2\phi_A}$.

Accordingly the conditional expected value of D_1 is $E(D_1(\omega,dB,B))=dB\frac{\Phi_A(B-dB)-\Phi_A(B+dB)}{\Phi_A(B_n+dB)+\Phi_A(B_n-dB)}\sim dB\frac{-2\langle\nabla\phi_A|dB\rangle}{2\phi_A}=-\langle\frac{\nabla\phi_A}{\phi_A}|dB\rangle dB$.

Overall the expected value is $E\big(D(\omega,B_n)\big)=\iint_{[0;1]^8}d\omega\;\;dB(\omega)\frac{\Phi_A(B_n-dB(\omega))}{\Phi_A(B_n+dB(\omega))+\Phi_A(B_n-dB(\omega))}$

$$\sim\iint dB\left(\frac{1}{2}-\frac{\langle\nabla\phi_A|dB\rangle}{2\Phi_A(B_n)}\right)=\frac{1}{2}\iint dB-\frac{1}{2\Phi_A(B_n)}\iint\left(\sum_i\partial_i\Phi\, db_i db_j\right)=-\frac{1}{2\Phi_A(B_n)}\sum_i\partial_i\Phi\iint\left(db_i db_j\right).$$

For symmetry reasons the integral resolves to a diagonal matrix as a multiple of the identity. I write

$$dB=\begin{pmatrix}db_0\\\vdots\\db_7\end{pmatrix}=[\![B_n]\!]\sin\Phi_A(B_n)^2q_0\begin{pmatrix}\cos(q_7)\\\sin(q_7)\cos(q_6)\\\vdots\\\sin(q_7)\ldots\sin(q_2)\cos(q_1)\\\sin(q_7)\ldots\sin(q_2)\sin(q_1)\end{pmatrix};\quad q_0\in[0;1],\quad q_i\in[0;\pi];\quad q_1\in[0;2\pi].$$

Then $E\big(D(\omega,B_n)\big)=-\frac{\nabla\Phi_A(B_n)}{2\Phi_A(B_n)}\iint\left(db_0(q_0,q_1)\right)^2D_8(q_1,\ldots,q_7)dq_0\cdots dq_7=$

$$-\frac{\nabla\Phi_A(B_n)}{2\Phi_A(B_n)}\iint\left(\left([\![B_n]\!]\sin\Phi_A(B_n)^2\right)^2q_0^2\frac{16}{5\pi}\cos(q_1)^2\sin(q_1)^6\right)dq_0\,dq_1=$$

$$-\frac{\nabla\Phi_A(B_n)}{2\Phi_A(B_n)}\left([\![B_n]\!]\sin\Phi_A(B_n)^2\right)^2\int_0^1q_0^2dq_0\frac{16}{5\pi}\int_0^\pi\cos(q_1)^2\sin(q_1)^6dq_1=-\frac{\nabla\Phi_A(B_n)}{6\Phi_A(B_n)}[\![B_n]\!]^2\sin\Phi_A(B_n)^4\frac{1}{8}.$$

The final relation $E(D(\omega,B))=-\frac{1}{48}[\![B]\!]^2\left(\sin\Phi_A(B)\right)^4\frac{\nabla\Phi_A(B)}{\Phi_A(B)}$ indicates that the mean values follow a gradient of a scalar field on S_B .

As yet this all happens within a 8 dimensional space. Let's see now how that maps to 3 dimensions and how dynamics may get defined.

2. Spacetime Geometry

2.1. Three Dimensional Space

The idea now is that all the dynamics above basically happen in 3 relevant dimension:

- Because of the linear mapping $B \to Bv$ only the 4 real dimensions of $S_B / \{B : Bv = 0\}$ remain that are relevant to the process.
- The scale of B should not be relevant too. Obviously for $\alpha \neq 0$ is $\Phi(\alpha B) = \Phi(B)$ and $\nabla\Phi(B) = \alpha\nabla\Phi(\alpha B)$.

The mapping $B \to Bv$ defines a linear transformation $T_v : \mathbb{R}^8 \to \mathbb{R}^4$. Let $\bar{B} \stackrel{\text{def}}{=} T_v B$. Then $[\![B]\!] = |\bar{B}|$ by directly applying its definition. The functions $\bar{D}(\omega, \bar{B}) \stackrel{\text{def}}{=} T_v D(\omega, B)$ and $\bar{\Phi}_A(\bar{B}) \stackrel{\text{def}}{=} \Phi_A(B)$ are obviously well defined.

Now Φ is a scalar and hence $\nabla\Phi$ transforms as a vector when changing coordinates. So because of the linearity of the transformation the relation $E(\bar{D}(\omega, \bar{B})) = -\frac{1}{48}|\bar{B}|^2 (\sin\bar{\Phi}_A(\bar{B}))^4 \frac{\nabla\bar{\Phi}_A(\bar{B})}{\bar{\Phi}_A(\bar{B})}$ still holds now taking the gradient along the new coordinates of $\bar{B}$.

Let $\bar{B} = |\bar{B}| R$ with $|R| = 1$. Then $E(\bar{D}(\omega, \bar{B})) = |\bar{B}| E(\bar{D}(\omega, R)) = -\frac{1}{48}|\bar{B}|^2 (\sin\Phi_A(R))^4 \frac{\nabla\Phi_A(|\bar{B}|R)}{\Phi_A(R)} = -\frac{1}{48}|\bar{B}| (\sin\Phi_A(R))^4 \frac{\nabla\Phi_A(R)}{\Phi_A(R)}$ and the relation $E(\bar{D}(\omega, R)) = -\frac{1}{48}(\sin\Phi_A(R))^4 \frac{\nabla\Phi_A(R)}{\Phi_A(R)}$ holds.

Taking polar coordinates $\bar{B}(q_0, \ldots, q_3)$, - $q_0 = |\bar{B}|$ - , keeps the relation above intact.

With $q = (q_1, q_2, q_3)$, $\Delta(\omega, q) \stackrel{\text{def}}{=} \bar{D}(\omega, \bar{B}(1, q))$ and $\Psi_A(q) \stackrel{\text{def}}{=} \bar{\Phi}_A(\bar{B}(1, q))$ we now have $E(\Delta(\omega, q)) = -\frac{1}{48}(\sin\Psi_A(q))^4 \frac{\nabla\Psi_A(q)}{\Psi_A(q)}$ and the according stochastic process now reads as $Q_{n+1} = \Delta(\omega, Q_n) + Q_n$ where the components of $Q_j \in \mathbb{R}^3$ are the 3 polar angles of some $\frac{B_j v}{[\![B_j]\!]}$.

This is exactly what I wanted to show: On one hand the given stochastic process satisfies the requirements to model state reduction. On the other hand the same process maps unambiguously in a well defined way to 3 spatial dimensions.

2.2. Average Movements

It is straightforward now to take the integer index of the stochastic series as a time variable. In order to get to smooth movements I have to consider mean values dr and d^2r with respect to a variable dt and expect some sort of movement equations. The mean speed is always zero $dr \stackrel{\text{def}}{=} E(D_0(\omega, B)) = 0$. However that is not true for $|dr|$ or dr^2 .

The following crude calculations will show where such an aproach may lead to. The presented toy model is definitely not a candidate for a final model. First calculate a mean square velocity $|dr|^2 \stackrel{\text{def}}{=} E(|dq|^2)$ [3]. Without loss of generality I may choose the perspective as $v = (1+i)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. The transformation $dB \to dBv$ then actually

3 To calculate the scalar product an norm in polar representation q is not just adding up and multiplying vector components. However you can choose the polar coordinates so that at the actual position q the product indeed gets calculated as that.

maps real vectors $B=\begin{pmatrix}b_0\\ \vdots\\ b_7\end{pmatrix}\rightarrow\begin{pmatrix}b_6-b_7\\ b_5-b_4\\ b_2-b_3\\ b_1-b_0\end{pmatrix}=b$. I first perform a orthonormal transformation $\begin{pmatrix}b_0\\ \vdots\\ b_7\end{pmatrix}\rightarrow\frac{1}{\sqrt{2}}\begin{pmatrix}b_6+b_7\\ \vdots\\ b_1+b_0\\ b_6-b_7\\ \vdots\\ b_1-b_0\end{pmatrix}$ so that b now results simply from a projection onto the latter 4 coordinates $B'=\begin{pmatrix}b_0\\ \vdots\\ b_7\end{pmatrix}\rightarrow\sqrt{2}\begin{pmatrix}b_4\\ b_5\\ b_6\\ b_7\end{pmatrix}=b$. Now As before I choose for the variation dB accordingly polar coordinates $(q_0,\ldots,q_7)$ as independent stochastic variables. With $|db|=\frac{1}{\sqrt{2}}|dB|\sin q_7\sin q_6\sin q_5\sin q_4$ this gives

$$db=\frac{1}{\sqrt{2}}|b|\sin(\Phi)^2 q_0\sin q_7\sin q_6\sin q_5\sin q_4\begin{pmatrix}\cos q_3\\ \sin q_3\cos q_2\\ \sin q_3\sin q_2\cos q_1\\ \sin q_3\sin q_2\sin q_1\end{pmatrix}$$

, where $q_0\in[0;1]$ is uniformly distributed.

So $\frac{|db|^2}{|b|^2}=|dq|^2=\frac{1}{2}\sin(\Psi)^4 q_0^2\sin q_7^2\sin q_6^2\sin q_5^2\sin q_4^2$ and

$$|dr|^2=E(|dq|^2)=\frac{1}{2}\int_0^1\sin(\Psi)^4 q_0^2\,dq_0\int_0^\pi d_7\sin(q_7)^2 dq_7\int_0^\pi d_6\sin(q_6)^2 dq_6\int_0^\pi d_5\sin(q_5)^2 dq_5\int_0^\pi d_4\sin(q_4)^2 dq_4=$$

$$\frac{1}{6}\sin(\Psi)^4\frac{16}{5\pi}\int_0^\pi\sin(q_7)^8 dq_7\frac{15}{16}\int_0^\pi\sin(q_6)^7 dq_6\frac{8}{3\pi}\int_0^\pi\sin(q_5)^6 dq_5\frac{3}{4}\int_0^\pi\sin(q_4)^5 dq_4=$$

$$\frac{1}{6}\sin(\Psi)^4\frac{16}{5\pi}\frac{35\pi}{128}\frac{15}{16}\frac{32}{35}\frac{8}{3\pi}\frac{5\pi}{16}\frac{3}{4}\frac{16}{15}=\frac{1}{12}\sin(\Psi)^4 .$$

2.3. An Emergent Spacetime

Lets consider a sufficiently large number of steps in the process projected along a given line. Then for the mean value the local condition $dr^2\le\frac{1}{12}\sin\Psi^4 dt^2$ holds. This obviously helps construct a pseudo metric with a line element $ds^2\overset{\text{def}}{=}\frac{1}{12}\sin\Psi^4 dt^2-dr^2\ge 0$. Now I can easily calculate the geodetic equations according to the related Riemannian space.

By means of the Lagrange function $L=\frac{1}{2}\left[\sin\Psi^4\dot{t}^2-12\sum_j\dot{r}_j^{\,2}\right]$ the Euler-Lagrange equation $\frac{d}{d\tau}\frac{\partial L}{\partial\dot{r}_j}-\frac{\partial L}{\partial r_j}=0$ results in $0=\frac{d}{d\tau}\left(-24\dot{r}_j\right)-4\cos\Psi\sin\Psi^3\partial_j\Psi\,\dot{t}^2$ or $\ddot{r}=-\frac{1}{6}\sin\Psi^4\dot{t}^2\frac{\nabla\Psi}{\tan\Psi}$ [6].

Because $\frac{dr}{dt}=0$ the relation $\frac{d^2r}{dt^2}=\frac{1}{\dot{t}^2}\ddot{r}-\frac{dr}{dt}\frac{1}{\dot{t}^2}\ddot{t}$ gives $\frac{d^2r}{dt^2}=\frac{1}{\dot{t}^2}\ddot{r}=-\frac{1}{6}\sin\Psi^4\frac{\nabla\Psi}{\tan\Psi}$. For small values of Ψ resp. q near an eigenvector of A is $\tan\Psi\simeq\Psi$. The result is proportional to the acceleration that the given stochastic process experiences for state changes.

However the mapping of matrices to states is not uniquely enforced by the GenI model. If I assume to have $\begin{pmatrix}b_0\\ \vdots\\ b_7\end{pmatrix}\rightarrow 2\begin{pmatrix}b_4\\ b_5\\ b_6\\ b_7\end{pmatrix}$ then the acceleration exactly meets the process dynamics $dr^2\le\frac{1}{24}\sin\Psi^4 dt^2$.

I now could argue that the actual state q of the system reflects the perspective of an observer. Indeed this position determines its degree of excitation and so some sort of movement.

At the position q I have the metric $g=\begin{pmatrix} \frac{1}{24}\left(\sin \Psi_A(q)\right)^4 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$. Now I transform locally the time coordinate $dt'=\frac{1}{2\sqrt{6}}\sin\Psi(q)^2\,dt$. Then the metric tensor resembles exactly the Minkowsky metric, i.e. the excitation is $\frac{dq^2}{dt'^2}\leq 1$. The local acceleration is $\frac{d^2 r}{dt'^2}=\nabla Z$ with a scalar field $Z_A(q)=2\ln(\Psi_A(q))$. Near an attractor resp. an projected eigenvector of A this could be interpreted as a movement along a geodesic line within the metric spacetime structure.

Obviously I could now determine the energy pulse tensor according to Einsteins field equation as an exercise. However I would not expect this to add significantly to the toy model for state reduction of a single spin ½ particle. At this point there is no clear understanding what movement or change really means, what the concept of an observer may be. Things actually may change when involving multi particle systems.[1]

3. Next Steps

The proposed model focuses solely on the measurement problem. It may be a first step to establish a new perspective on QM and GR. As a simple toy model it is still far from finally explaining how GR really emerges from a quantum model. One issue among others is that state rotations are not reflected as such in the 3 dimensional space. So this toy model is definitely not a true candidate for a final model.

At first sight a better suited representation in space is given by $R:\mathbb{C}^2\rightarrow\mathbb{R}^3; b\rightarrow\begin{pmatrix}\langle p_z b|b\rangle\\ \langle p_y b|b\rangle\\ \langle p_x b|b\rangle\end{pmatrix}$. This map is linear in the Pauli matrices and hence transforms like a vector. I will rather consider this one in future while proceeding to a final model. At this point it will not add substantially to the principles shown here but on the other hand adds a lot of complexity.

From my point of view the approach looks promising enough to continue with the next steps: investigating in multi particle systems resp. spin > ½ and in the capability of a state to operate on itself. The picture I have in mind is a universe undergoing continuous state reduction while operating on itself as a measuring device. I will give here only a brief overview on the next step[4].

To extent the model to many particles it seems straightforward to leverage the tensor representation. Classically a system of two particles with states $b, c\in\mathbb{C}^2$ gets represented by $b\otimes c$. Within the GenI Model the states B and C may be combined as $B\otimes C$. With a perspective v this results in the classical state $(B\otimes C)(v\otimes v)=Bv\otimes Cv$. The state reduction process can easily utilize these definition to calculate the required scalar products, angles and the scalar field. This is completely OK for the external view. The problem now is the feedback process. An empirical approach may start to vary the tensor parameters as it is possible in the 2-dimensional case. However simulations show that this does not work. The process does not even converge over time except in rare cases.

What effectively may work is to take a view of appropriate organized sets of individual particles. The issue here is that the external view as a conventional state may get resembled by many different sets of particles. All these different resembles must not change the external features of the state reduction process. However the approach to take on more complex aggregates is not obvious at all. First attempts suggest that concepts of competition and cooperation of particles should be part of such a model. So looking at swarm behavior and organization may provide more insight in feasible approaches.

4 Ask a question to the universe and you start a process with a big bang, running billions of years involving trillions of entities. But that does not mean anything to you who may experience an immediate answer as simple as “42” .

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