

# OPERATIONAL PATTERNS IN QUANTUM STATES

SIEGFRIED GENREITH

ABSTRACT. A major issue for modern physics is how reality in terms of general relativity may emerge from quantum mechanics.

Two observations motivate this paper:

1) General relativity (GR) with Einstein's Field equation is highly recursive in how it is formulated. The energy distribution determines spacetime geometry and vice versa spacetime geometry determines local trajectories and the evolution of the mass-energy distribution. Such a feature is absent in quantum mechanics (QM). Here the systems state and observables are different things. A state cannot operate on itself. To introduce recursion into the core of QM requires an extended concept of what the state of a system is.

2) Many papers on collapse or decoherence deal with how observed classical reality is determined by QM. Common understanding is that the Universe evolves according to the Schroedinger equation - i.e. within a unitary U-process. I don't know of any work that considers the Universe - as we observe it - evolving according to a badly understood state reduction process (R-process).

This paper proposes an extension of the QM standard model to address both concerns. It enables two realities linked by their respective symmetries. Its final purpose is to enable state reduction dynamics to be defined as a simple random walk near process .

## INTRODUCTION

The proposal in this paper has initially been motivated when looking for proceedings in artificial intelligence during the past decades. Though recent models and implementations perform quite successful on complex tasks the gaps are obvious. The understanding of intelligent reasoning and problem solving has not fundamentally improved over the past 20 years. A similar situation can be found in biology and psychology. They also fail to explain what really goes on. Still insufficient but yet most rigorous are attempts to explain intelligent behavior (and finally how consciousness may evolve) by quantum models.

After investigating a bit in a top-down model for decision taking this is a first attempt to translate some ideas into mathematics and apply some of the findings to physics.

When investigating in a reasonable modeling approach it appears that base quantum models together with wave reduction make sense in modeling conscious - or intelligent - decision taking. This works basically by translating quantum concepts of state, decoherence/collapse, operator, eigenvalues, eigenvectors to terms that make more sense to psychologists.

Of course there is an obvious issue in applying quantum models in this arena: Here one deals with interactions of two or more basically equivalent entities. The interaction may also work vice versa. This is not true for the standard model of decoherence that deals with operators and their intrinsic symmetries on the one hand and clearly separated states on the other with very different degrees of freedom. Therefore we probably need to consider an extended concept of what a state is.

On the other hand this all seems to deal with the question of reality. How do classical reality - dominated by gravity and general relativity - and quantum reality - characterized by the Schrödinger equation and the collapse process - correlate? Some authors support the idea that gravity in QM cannot be understood without a deeper insight into the wave reduction process.[1] This point of view leads to two observations:

1. General Relativity with Einstein's Field equation is highly recursive in how it is formulated. The energy distribution determines spacetime geometry and vice versa spacetime geometry (operator) determines local trajectories and the evolution of the mass-energy distribution (state). This feature is absent in quantum mechanics. Identifying states with operators fails alone because of different degrees of freedom. To introduce recursion in QM requires an extended concept of states.
2. Many papers on collapse or decoherence deal with how observed classical reality is determined by QM. Common understanding is that the Universe evolves according to the Schroedinger equation – i.e.

within a unitary U-process. I don't know of any work that considers the Universe - as we observe it - evolving according to a badly understood R(eduction)-process.

But what could be a hidden structure within the state of a quantum particle? There is nothing in the standard model that could answer such a question. Each attempt to artificially extend a state will certainly look ridiculous.

On the other hand it is obvious that you can embed operational information in quantum states[2]. In quantum computing engineers are going to code operations and data in state superpositions of spin-  $\frac{1}{2}$  particles. They then need an artificial device - the computer - to get detected and finally executed. The approach in this paper is different in so far that it is looking for intrinsic information that may not need any artificial device to get executed. It deals with self referentiality of formal structures.

The final section describing a symmetric model for decoherence will show a door that is not there in the standard model. Further investigations may improve present understanding of the roles of QM, GR and intelligent decision taking within reality.

The approach obviously needs to redefine what a state is.

- From a very basic point of view the definition should be discrete - i.e. integer based.[3]
- As quantum observables in the standard model always are hermitian operators and hence deal with real eigenvalues the definition should probably be based on real integers.
- The internal structure of such a state should evolve out of symmetry arguments and
- finally the standard model should emerge by simple principles.

So this is exactly the starting point here. Let's clear memories and start to forget almost everything about modeling in quantum physics. Leading to a revised modeling approach for states and operators on spin- $\frac{1}{2}$  particles the model should explain the role of integer, real or complex numbers and the Pauli spin matrices and not taking them as given or introduce any of them by reference to classical physics.

So this is what you should expect in the following.

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### 1. A JOURNEY TO INTEGER SYMMETRIES

Let's start a journey to provide some motivation and justification for the following ideas.

The assumption is – while looking for symmetries, creating symmetries and breaking symmetries - : “Something is”.

The existence of one “Something” probably means there may be two or three or more. Creating the free abelian group on “Something” introduces  $\mathbb{Z}$  to the model. Its common ring structure is straightforward.

Next step is to introduce dimensionality.  $\mathbb{Z}^2$  gets to a non-decomposable  $\mathbb{Z}$ -algebra by defining  $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ , i.e. isomorphic to the commutative Dedekind-ring  $\mathbb{Z}[i]$  of complex integers.[4] The group of units is given by four elements  $\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ \pm 1 \end{pmatrix}$  resp.  $\{1, i, -1, -i\}$ .

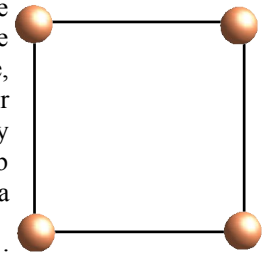
You could take it from a different p. o. v. by looking at the geometric transformations of a line within a 1-dimensional space where the line ends are represented by integer values.



That is the reflection  $D_2$  exchanging the 1<sup>st</sup> with the 2<sup>nd</sup> element. Then extend this cyclic group of order 2 with two operations  $s_0, s_1$  switching the sign of the 1<sup>st</sup> resp. 2<sup>nd</sup> vector element. Based on this Group  $G_l$  of order 8 generated by  $(D_2, s_0)$  I try to figure out a representation of a subgroup  $U$  in  $G_l$  by elements in  $\mathbb{Z}^2$  i. e.  $U \subset G_l$ ,  $T \subset \mathbb{Z}^2$ ,  $U \times T \rightarrow T$  and a bijection  $\phi: T \rightarrow U$  with the property  $\phi(us) = u \phi(s)$ . This would then induce a group structure on  $T$  (see next chapter below). Therefore I have to choose a subgroup and a 1-element in  $\mathbb{Z}^2$  whose orbit has the same cardinality as the subgroup. There are a few equivalent alternatives to do so. Select the cyclic subgroup  $U = \langle D_2, s_1 \rangle$  of order 4, then the orbit  $\mathfrak{N}(U \begin{pmatrix} 1 \\ 0 \end{pmatrix}) = \mathfrak{N} U$  matches the group order and I can identify  $D_2 s_1$  with  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  resp. the complex  $i$ . The order of the subgroup is twice the  $\mathbb{Z}$ -dimension of  $\mathbb{Z}^2$ .

Now look at the tensor product  $\mathbb{Z}^2 \otimes \mathbb{Z}^2$ . The natural  $\mathbb{Z}$ -algebra structure is given by the ring  $Mat_{\mathbb{Z}}(2 \times 2)$ . The group of orthogonal transformations is given by the matrices  $T$  with  $|det T| = 1$ . These are 8 reflections and rotations on  $\mathbb{Z}^2$  as there are  $\begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$ .

Now imagine the bilinear tensors on  $\mathbb{Z}^2$  as vectors with 4 elements. The most symmetric figure on a 2-dimensional plane is a square because all its nodes are equivalent. Its nodes are represented by integers. Now think about their geometric features. A symmetry group may get build by the symmetries  $O(2)$  of a geometric square, together with the operations that simply inverse the sign of one node of the tensor  $G_2 = \langle D_4, S, s_0 \rangle$  where  $D_4$  rotates the elements clockwise,  $S$  reflects bottom to top and  $s_0$  changes the sign of the top left element. Hence its order is  $8 * 2^4 = 2^7$ . If you analyze its structure nothing really new will appear. If you identify the group identity with the matrix identity then the approach above leads to a sub algebra generated by the orthogonal matrices above that represent the elements of a selected subgroup  $U = \langle S, S s_2 s_3 \rangle$  and a relation  $S \sim \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  and  $S s_2 s_3 \sim \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \sim i$ .



Again this is a non-decomposable algebra and in this case the algebra structure is that of the tensor algebra. The order of the subgroup in this case is  $8 =$  twice the  $\mathbb{Z}$ -dimension of  $\mathbb{Z}^2 \otimes \mathbb{Z}^2$ .

The other way around reads like this: A full matrix ring  $Mat_K(n \times n)$  on any field  $K$  may be represented by operating on elementary matrices  $M_{kl} \stackrel{\text{def}}{=} (\delta_{ik} \delta_{jl})$  as a base in  $K^{n^2}$ . In some sense now the vectors in terms of this base code their own operations. By this requirement this base and any of its permutations are exposed from any other. Furthermore any irreducible algebra with minimal condition for ideals is isomorphic to a full matrix ring on a skew field. In all these cases I trivially can construct such a base.

There is some plausibility in the assumption that an extended model for spin  $\frac{1}{2}$  particles should have 8 real degrees of freedom instead of 4. Actually there is no reason to doubt that operators as observables in the standard model are insufficient to completely explain a quantum measurement. States should be able to operate on each other quite the same way matrices do – thus coding their own operations.

Now dealing with integers no minimal condition for ideals in a integers based algebra holds. And with 8 vector components there is no straightforward way to achieve a algebra structure.

## 2. INTRODUCING META INFORMATION

So now look at a model providing 8 integer degrees of freedom. Obviously these 8 integers cannot easily be put to a 2-dimensional structure to get something like matrices. The geometric form with 8 nodes that does not expose any node among the others is the cube.

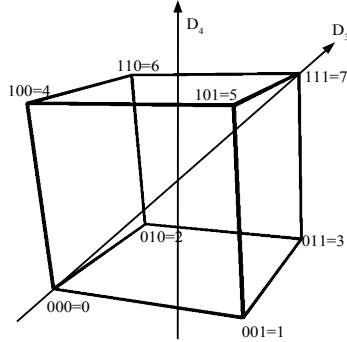


FIGURE 1: STATE CUBE

### 2.1. CUBE ARITHMETICS

**Definition 2.1.** I now take a state as an element in  $\mathbb{Z}^8$  and consider it as cubes with integers at each node. I call it the state grid  $A_G$  and write its elements as  $\begin{bmatrix} b_0 & b_1 \\ b_2 & b_3 \end{bmatrix} \begin{bmatrix} b_4 & b_5 \\ b_6 & b_7 \end{bmatrix}$ . I select the Euclidean product

$\langle A|B \rangle \stackrel{\text{def}}{=} \sum_{i=0}^7 a_i b_i$  as the canonical scalar product on  $A_G$ . That means I assume the axes to be orthogonal and the grid distance to be 1.

I look for an appropriate coding that may give rise to operational information and allows them to operate on themselves. Operating states together with the usual element wise addition then form a  $\mathbb{Z}$ -algebra within the state grid. Within this context then the grid as a  $\mathbb{Z}$ -module with canonical base is exposed from any other. What I'm basically looking for is some sort of a matrix operation.

Something straightforward to manipulate a cube is to do it by its symmetry group. I choose the orthogonal group  $O(3)$  extended by the operations that invert the sign of the integer at a node. Hence its order is  $\aleph G = 48 \cdot 2^8 = 3 \cdot 2^{12}$ . It turns out that its structure provides some useful features. I count the nodes as indicated above starting with 0. Let  $D_3$  be the rotation about the 0-7 axis,  $D_4$  the rotation about the axis through the respective centers of the 0-1-2-3 and the 4-5-6-7 squares – both counter clockwise as seen from the top of the arrow -,  $S$  be the reflection exchanging the top and bottom square, and let  $s_i$  invert the sign of the  $i^{\text{th}}$  node.

General operations on the 8-dimensional  $\mathbb{Z}$ -module given by the group  $G_3 \stackrel{\text{def}}{=} (D_3, D_4, S, s_0, \dots, s_7)$ . The problem now reads:

Find a maximal subgroup  $U \subset G_3$  and a subset  $T \subset A_G$  such that  $\exists t \in T: Ut = T$ .  $T$  then generates an 8-dimensional  $\mathbb{Z}$ -submodule  $A_T \subset A_G$ .  $U$  may be identified with  $T$  generating  $A_T$  as a  $\mathbb{Z}$ -algebra.

Given the operation  $G_3 \times A_G \rightarrow A_G$  I look out for  $U \subset G_3$ ,  $T \subset A_G$ ,  $U \times T \rightarrow T$  and a bijection  $\phi: T \rightarrow U$  with the property  $\phi(us) = u\phi(s)$ .

Let  $T_0 = \phi^{-1}(e)$ ,  $T_u = uT_1 = \phi^{-1}(u)$  and hence  $T = UT_0$ .

What these conditions actually tell is that  $A_T$  will be a faithful representation of  $U$ . But at the moment  $U$  as well as the algebra structure is still to be determined. In the real complex environment the irreducible representations of a given group can all be determined by the regular one. That is however not necessarily correct if I consider integer modules.

Given the conditions above this mapping then induces an isomorphism between  $U \times T \rightarrow T$  and the operation of  $U$  on itself. Vice versa  $T$  inherits operational features from  $U$  by  $s \circ t \stackrel{\text{def}}{=} \phi(s)t = \phi^{-1}(\phi(s)\phi(t))$ . This induces an algebra structure on  $A_T$ .

Now I give some assumptions about  $G$ 's cardinality. It must be at least  $8 = \dim_{\mathbb{Z}}(A_G) \leq \aleph G$ . If  $\aleph T = \aleph U = 8$  then  $T \neq -T$  and  $T \cap -T = \emptyset$  because the elements of  $T$  are linear independent. You can extend  $T$  by  $-T$  and accordingly  $U$  by the operation  $u_{-1} \stackrel{\text{def}}{=} s_0 \dots s_7 \in \text{center}(G_3)$  that multiplies each vector component by  $-1$  and thus extending  $U$  to cardinality 16.

I'm now going to construct a subgroup  $U \in G_3$  of order 16. Then I have to find a vector  $T_0$  whose trace has the same size as  $U$  so that  $U T_0$  represents  $U$  in  $A_T \subset A_G$  holding the equations above.

I'll shortcut the attempts to find a pair  $(U, T)$  and start over with a subset  $U$  of  $G_3$ . (It is a bit like solving Sudoku puzzles.) I choose a group  $G$  generated by elements  $(D_4, S, s_0)$  operating on cubes  $B = \begin{bmatrix} b_0 & b_1 \\ b_2 & b_3 \end{bmatrix} \begin{bmatrix} b_4 & b_5 \\ b_6 & b_7 \end{bmatrix}$  in the following way:

$$\begin{aligned} \bullet \quad D_4 B &= \begin{bmatrix} b_2 & b_0 \\ b_3 & b_1 \end{bmatrix} \begin{bmatrix} b_6 & b_4 \\ b_7 & b_5 \end{bmatrix} \\ \bullet \quad S B &\stackrel{\text{def}}{=} \begin{bmatrix} b_4 & b_5 \\ b_6 & b_7 \end{bmatrix} \begin{bmatrix} b_0 & b_1 \\ b_2 & b_3 \end{bmatrix} \\ \bullet \quad s_0 B &= \begin{bmatrix} -b_0 & b_1 \\ b_2 & b_3 \end{bmatrix} \begin{bmatrix} b_4 & b_5 \\ b_6 & b_7 \end{bmatrix} \end{aligned}$$

$D_4$  (rotation) is of order 4,  $S$  (reflection) and  $s_0$  are of order 2. The  $s_i, i=0..7$ , are defined accordingly.  $D_4$  and  $S$  represent cube symmetries. The order of that subgroup is  $\aleph G = 8 \cdot 2^8 = 2^{11}$  and will get reduced further.

**Proposition 2.1.** Let  $B_i \in G$  be defined by  $B_0 \stackrel{\text{def}}{=} D_4^2 s_0 s_1 s_4 s_5$ ,  $B_1 \stackrel{\text{def}}{=} S s_4 s_5 s_6 s_7$ ,  $B_2 \stackrel{\text{def}}{=} s_0 s_1 s_2 s_3$ . Then  $U = (B_0, B_1, B_2) \subset G_3$  is a subgroup of order 16.  $U$  is isomorphic to the group generated by the Pauli spin matrices. .

The subgroup  $(B_0)$  of order 4 is the center of  $U$ . The Pauli matrix equivalents are then identified to be



$$\begin{aligned} \bullet \quad p_1 &\stackrel{\text{def}}{=} B_2 B_1 = S \\ \bullet \quad p_2 &\stackrel{\text{def}}{=} B_1 B_0^3 = D_4^2 S s_2 s_3 s_4 s_5 \\ \bullet \quad p_3 &\stackrel{\text{def}}{=} B_2 = s_0 s_1 s_2 s_3 \end{aligned}$$

By direct calculation you can validate all the relationships for the Pauli group.

Proceeding further we now have to determine a suitable set of 16 vectors that may inherit  $U$ 's operational characteristics. It is sufficient to identify a candidate for the identity  $T_1 \in T$  first and then calculate all other element. There are a couple of restrictions to the components of  $T_0 = [t_i]$  that result from the requirement that the trace of  $T_i$  under  $G$  is  $T$ .

1. For  $p_3 T_1 \neq T_1$  the condition  $\exists t \in \{t_0 t_1 t_2 t_3\} : t \neq 0$  holds.
2. The condition  $p_1 T_1 \neq T_1$  implies  $\exists i \leq 3 : t_i \neq t_{i+4}$ .
3. The common denominator  $d$  of the  $t_i$  must be 1. Otherwise  $T_0 = d \cdot \bar{T}_0$  cannot be the identity operator.

As a perfect starting point turn out those 32 cubes, having only  $0, \pm 1$  at its nodes in a way that if one node is 0 then all its neighbors aren't and vice versa.

I call these cubes  $T^+$  type cubes  and  $T^-$  type cubes . The spheres represent  $\pm 1$  each. Their representations are  $T^+ \stackrel{\text{def}}{=} \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix}$  and  $T^- \stackrel{\text{def}}{=} \begin{bmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{bmatrix} \begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix}$ .

I may choose any of these as the identity  $T_0$ . They all meet  $U T_0 = T$ . Choosing a different  $T_0$  results in a orthonormal base transformation. Without loss of generality I may choose  $T_0 \stackrel{\text{def}}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . As you see

below it results in a set  $T$  of the 16 vectors with an even number of -1s, i.e. 0, 2 or 4 values equal -1. It obviously meets  $T=-T$ . Here are the explicit mappings:

$U$ symbolic	(Pauli) matrix equivalent	Operation on $A_G$	Representation in $A_G$
$e \sim 1$	$Id = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$id_{A_G}$	$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$(p_1 p_2 p_3)^2$ $= p_{-1} \sim -1$	$-Id = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$B_0^2 = s_0 s_1 s_2 s_3 s_4 s_5 s_6 s_7$	$T_{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
$p_1 p_2 p_3$ $= p_i$	$i Id = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$	$B_0 = D_4^2 s_0 s_1 s_4 s_5 = (D_4 s_1 s_5)^2$	$T_i = T_4 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
$p_3 p_2 p_1$ $= p_{-i}$	$-i Id = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$	$B_0^3 = D_4^2 s_2 s_3 s_6 s_7$	$T_{-i} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
$p_3$	$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$s_0 s_1 s_2 s_3$	$T_{p_0} = T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$p_2$	$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$D_4^2 S s_2 s_3 s_4 s_5$	$T_{p_1} = T_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
$p_1$	$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$S$	$T_{p_2} = T_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$p_3 p_2$	$i \sigma_1 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$	$D_4^2 S s_2 s_3 s_6 s_7$	$T_{p_0 p_1} = T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
$p_3 p_1$	$i \sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$S s_4 s_5 s_6 s_7$	$T_{p_0 p_2} = T_6 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$p_2 p_1$	$-i \sigma_3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$	$D_4^2 s_0 s_1 s_6 s_7$	$T_{p_1 p_2} = -T_7 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Table 1: GenI Reference Table

This subset  $T = \{T_\alpha : \alpha \in U\}$  of 16 cubes generate the  $\mathbb{Z}$ -algebra  $A_T$  with multiplication induced by the group operation of  $U$ . The first 4 elements above form the center of  $U$ , hence the generated sub algebra is the 2-dimensional center  $K_T$  of  $A_T$ . The addition is realized component wise as well as multiplication by integers.

Without loss of generality the cubes  $T_j : j \in \{0, \dots, 7\}$  in table 1 will be selected as a  $\mathbb{Z}$ -base for the algebra in the following calculations. The specific choice is not important because a different choice will mean a permutation and multiplication of base vectors by  $\pm 1$ . This is equivalent to performing a orthonormal transformation whose determinant must be a unit in  $\mathbb{Z}$ , i.e.  $\pm 1$ . Such transformations do not change the Euclidean scalar product and norm.

## 2.2. TRANSFORMATIONS

Let  $E_i$  denote the canonical base for the state grid  $\mathbb{Z}^8$ . This base determines the scalar product within the state grid. The transformation matrix into the base  $(T_0, \dots, T_7)$  of  $A_T$  is given by

$$M = \begin{pmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \text{ mapping } E_i \rightarrow T_i. \text{ The transformation is orthogonal keeping}$$

angles intact. Its inverse  $M^{-1} = \frac{1}{4} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \end{pmatrix}$  can get defined on  $A_T$ . However  $\frac{1}{4}$  is

not an integer indicating that  $A_T$  is a true subset of  $A_G$ . So to get it mathematical right I have to localize  $\mathbb{Z}$  as  $\mathbb{Z}_2 \stackrel{\text{def}}{=} \mathbb{Z}[\frac{1}{2}]$ . The canonical isomorphism  $A_T \rightarrow \text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$ ;  $p_j \rightarrow \sigma_j$  extends to

$$\Psi: A_G \rightarrow \text{Mat}_{\mathbb{Z}[i]}: \begin{pmatrix} b_0 \\ \vdots \\ b_7 \end{pmatrix} \rightarrow \frac{1+i}{2} \begin{pmatrix} b_5 - i b_6 & b_4 - i b_7 \\ b_1 - i b_2 & b_0 - i b_3 \end{pmatrix}.$$

Let  $\bar{A}_G = \Psi(A_G)$  and  $\bar{A}_T = \Psi(A_T)$ . Then  $\bar{A}_T$  is generated by  $(Id, \sigma_1, \sigma_2, \sigma_3, iId, i\sigma_1, i\sigma_2, i\sigma_3)$  as a  $\mathbb{Z}$  base. A canonical  $\mathbb{Z}$  base for the full matrix ring is  $\left( \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}, \dots \right)$ . The transformation between the

bases is given by  $X = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$ . This is again a

orthogonal transformation but defining a different grid.

The induced scalar product on  $\text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$  taken as vectors is  $\langle a|b \rangle = 2\Re(\sum_{ij} \bar{a}_i b_j)$ . This is double the real part of the Hilbert product on  $\mathbb{Z}[i]^4$

.<sup>1</sup> So in  $\bar{A}_G$  the angle  $\phi$  between  $Id$  and  $E_{11}$  is  $\cos \phi = \frac{\langle Id|E_{11} \rangle}{|Id||E_{11}|} = \frac{2}{2 \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$  and  $\phi = \frac{\pi}{4}$ . All the other angles are either this or  $\frac{\pi}{2}$ . The grid width is  $\sqrt{2}$ . This corresponds to a square grid extended to one with a diagonal orientation.

On the other hand in  $A_G$  the angle  $\psi$  between  $Id$  and  $E_1$  is  $\cos \psi = \frac{\langle Id|E_1 \rangle}{|Id||E_1|} = \frac{1}{2 \cdot 1} = \frac{1}{2}$  and  $\psi = \frac{\pi}{3}$ . This angle corresponds to a 4-dimensional cube grid extended along the 4 cube diagonal lines. This is not a straightforward thing to consider within  $\text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$ .

We will see in the following what impact this will have on the algebra structure.

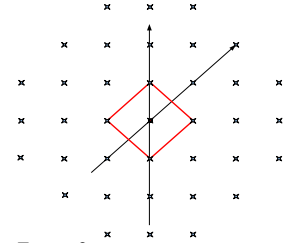


FIGURE 2: EMBEDDING A SQUARE GRID

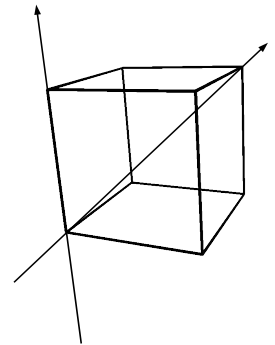


FIGURE 3: EMBEDDING A CUBE GRID

<sup>1</sup> This may be the source for the weird factor 2 that is omnipresent in classical QM.



### 3. THE GENI MODEL

#### 3.1. ALGEBRA STRUCTURE

**Definition 3.1.** A GenI algebra is a triple  $(A_T, \circ, \langle \cdot | \cdot \rangle)$  of a  $\mathbb{Z}$ -submodule  $A_T \subset \mathbb{Z}^8$  generated by the  $(T_j: j \in \{0, \dots, 7\})$ , an operation  $\circ: A_T \times A_T \rightarrow A_T$  defined by

$$2 \begin{pmatrix} a_0 \\ \vdots \\ a_7 \end{pmatrix} \circ \begin{pmatrix} b_0 \\ \vdots \\ b_7 \end{pmatrix} = \begin{pmatrix} a_0(b_0+b_3)+a_1(b_4+b_7)+a_2(b_4-b_7)+a_3(b_0-b_3) \\ a_0(b_1+b_2)+a_1(b_5+b_6)+a_2(b_5-b_6)+a_3(b_1-b_2) \\ a_0(b_2-b_1)+a_1(b_6-b_5)+a_2(b_5+b_6)+a_3(b_1+b_2) \\ a_0(b_3-b_0)+a_1(b_7-b_4)+a_2(b_4+b_7)+a_3(b_0+b_3) \\ a_4(b_0+b_3)+a_5(b_4+b_7)+a_6(b_4-b_7)+a_7(b_0-b_3) \\ a_4(b_1+b_2)+a_5(b_5+b_6)+a_6(b_5-b_6)+a_7(b_1-b_2) \\ a_4(b_2-b_1)+a_5(b_6-b_5)+a_6(b_5+b_6)+a_7(b_1+b_2) \\ a_4(b_3-b_0)+a_5(b_7-b_4)+a_6(b_4+b_7)+a_7(b_0+b_3) \end{pmatrix} \text{ and the canonical Euclidean scalar product on } \mathbb{Z}^8.$$

**Proposition 3.1.** The GenI Algebra  $A_T$  is a non decomposable representation of the Pauli Group. Furthermore it is not decomposable into a direct sum of left  $A_T$  ideals and there is no idempotent element except  $e$ .

*Proof:* The first part of the statement is clear by the construction of the algebra and the mapping  $p_j \rightarrow T_j$ .

Let  $A_T = L_1 + L_2$  and  $e = e_1 + e_2$  be the according decomposition of the identity into idempotents. Let  $e_1 = c_1 + c_1 p_1 + c_2 p_2 + c_3 p_3$ ,  $c \neq 0$ ,  $c, c_j \in \mathbb{Z}[p_i] \sim \mathbb{Z}[i]$ . Because  $p_k p_j = -p_j p_k$ ; for  $k \neq j$ , we get  $e_1 = e_1^2 = c^2 + c_1^2 + c_2^2 + c_3^2 + 2c c_1 p_1 + 2c c_2 p_2 + 2c c_3 p_3$  and hence by direct comparison  $2c c_j = c_j$ . As we deal with an integrity ring and integers this means  $c_j = 0$  and hence  $c = 1$ .

If I extend the coefficient ring  $\mathbb{Z}$  to rational (and finally real) numbers  $\mathbb{Q}$  then  $A_G$  and  $A_T$  both lead to the ring  $A_R \simeq \mathbb{Q} \otimes A_T$ .  $A_T$  is a non-maximal order in  $A_R$ . [5]

By extending the isomorphism  $\Psi: A_G \rightarrow \text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$  to  $\mathbb{Q}$  the ring  $A_R$  is isomorphic to the full matrix ring  $\text{Mat}_{\mathbb{Q}[i]}(2 \times 2)$  and hence simple. It splits into two minimal left ideals  $L_1$  and  $L_2$  represented by the two columns of the matrix ring and hence  $A_R = L_1 \oplus L_2$ .

The given isomorphism  $\Psi: A_R \simeq \text{Mat}_{\mathbb{Q}[i]}(2 \times 2)$  maps these ideals exactly onto the two isomorphic  $A_R$  ideals

$$L_1 = \left[ \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix} \begin{pmatrix} 0 & a_5 \\ a_6 & 0 \end{pmatrix} \right] : a_i \in \mathbb{Q} \text{ and } L_2 = \left[ \begin{pmatrix} a_0 & 0 \\ 0 & a_3 \end{pmatrix} \begin{pmatrix} a_4 & 0 \\ 0 & a_7 \end{pmatrix} \right] : a_i \in \mathbb{Q}.$$

The  $L_i$  enable equivalent irreducible faithful representations of  $A_R$ . Hence  $A_R$  is a primitive ring. The according decomposition of the 1-element is (symbolical)  $1 = \frac{1}{2}(1 + p_0) + \frac{1}{2}(1 - p_0)$ .

Any minimal left  $A_R$  module  $M$  is isomorphic to  $L_i$ . Now define the  $A_R$  module  $\mathbb{Q}[i]^2$  by  $A v \stackrel{\text{def}}{=} \Psi(A) v$ . Given any  $A_R$ -homomorphism  $\zeta: A_R \rightarrow \mathbb{Q}[i]^2$  the kernel  $K_\zeta$  is invariant under  $A_R$  and a left ideal. Hence if  $\zeta \neq 0$  then  $K_\zeta$  is isomorphic to  $L_2$  and  $\zeta$  induces a isomorphism of  $A_R / L_2 \simeq L_1 \rightarrow \mathbb{Q}[i]^2$ .

Back to the integer case the ideals above map to the left  $A_T$  modules  $A^- = L_1 \cap A_G$  and  $A^+ = L_2 \cap A_G$  in  $A_G = A \oplus A^+$ .

For completeness I mention the following

**Proposition 3.2.** There is a algebra monomorphism  $\Psi: A_T \rightarrow \text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$  given by

$$2 \Psi \left( \begin{pmatrix} b_0 \\ \vdots \\ b_7 \end{pmatrix} \right) = (1+i) \begin{pmatrix} b_5 - i b_6 & b_4 - i b_7 \\ b_1 - i b_2 & b_0 - i b_3 \end{pmatrix} \text{ that is well defined on } A_T. \text{ The induced scalar product on } \text{Mat}_{\mathbb{Z}[i]}(2 \times 2)$$

is  $\langle (a_{ij}) | (b_{ij}) \rangle = 2 \Re \left( \sum_{ij} \bar{a}_{ij} b_{ij} \right)$  - two times the real part of the Hilbert product on  $\mathbb{Z}[i]^4$ .

### 3.2. MODULE STRUCTURE AND CLASSICAL STATES

The question here is how classical states may evolve out of this model. The irreducible left ideals of the algebra should provide the means to get there.

Let's look at the structure of  $A_G$  as an  $A_T$ -module.  $E_6$  and  $E_7$  each determine a 4-dimensional representation on invariant subspaces as  $A^+ = A_T[E_7] = \mathbb{Z}[E_0, E_3, E_4, E_7]$  and  $A^- = A_T[E_6] = \mathbb{Z}[E_1, E_2, E_5, E_6]$  respectively.

**Proposition 3.3.** *There is a exact sequence  $0 \rightarrow K_\zeta \rightarrow A_G \xrightarrow[\rho]{\zeta} \mathbb{Z}[i]^2 \rightarrow 0$ . Given the  $A_T$  homomorphisms  $\zeta: A_G \rightarrow \mathbb{Z}[i]^2$  there exists a map  $\rho: \mathbb{Z}[i]^2 \rightarrow A_G$  with  $\zeta \circ \rho = id_{\mathbb{Z}[i]^2}$ . The images  $\zeta(E_6)$  and  $\zeta(E_7)$  each generate  $\mathbb{Z}[i]^2$  as an  $A_T$ -Module. There are exactly 4 non-equivalent<sup>2</sup> mappings  $\zeta^{(\beta)}$  given by  $\zeta^{(\beta)}(E_6) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\zeta^{(\beta)}(E_7) = \beta \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  where  $\beta \in \{\pm 1; \pm i\}$ . With  $\alpha \stackrel{\text{def}}{=} \beta^{-1}$  two reverse mappings are  $\rho_0^{(\alpha)} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = \alpha \rho_0 \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = \alpha E_7$  and  $\rho_1 \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = E_6$ .*

*Proof:* Any  $A_T$  homomorphism  $\zeta: A_G \rightarrow \mathbb{Z}[i]^2$  is determined by  $\zeta(E_7)$  and  $\zeta(E_6)$ . Vice versa any  $A_T$  homomorphism  $\rho: \mathbb{Z}[i]^2 \rightarrow A_G$  is completely defined by  $\rho \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)$ .

Because the  $\zeta^{(\beta)}(E_j)$  each generate  $\mathbb{Z}[i]^2$  this leads to  $\zeta(E_6), \zeta(E_7) \in \left\{ \begin{pmatrix} \beta \\ 0 \end{pmatrix} : \beta \in \{\pm 1; \pm i\} \right\}$ . So as the mappings are  $\mathbb{Z}[i]$ -linear I may choose without restriction of generality  $\zeta^{(\beta)}(E_6) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\zeta^{(\beta)}(E_7) = \begin{pmatrix} \beta \\ 0 \end{pmatrix}$ .

Reversely  $\rho \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \in (\zeta^{(\beta)})^{-1} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = \{E_6, \alpha E_7\}$ . It is  $\zeta^{(\beta)}: A_G \rightarrow \mathbb{Z}[i]^2; A \rightarrow \begin{pmatrix} a_6 + \beta a_7 + i(a_5 + \beta a_4) \\ a_2 + \beta a_3 + i(a_1 + \beta a_0) \end{pmatrix}$  and  $\zeta^{(\beta)} \circ \rho_1 = \zeta^{(\beta)} \circ \rho_0^{(\alpha)} = id_{\mathbb{Z}[i]^2}$ . With  $\vec{s} = \begin{pmatrix} s_0 + is_1 \\ t_0 + it_1 \end{pmatrix}$  both mappings  $\rho_0(\vec{s}) = \begin{bmatrix} t_1 & 0 \\ 0 & t_0 \end{bmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_0 \end{bmatrix}$ ,  $\rho_1(\vec{s}) = \begin{bmatrix} 0 & t_1 \\ t_0 & 0 \end{bmatrix} \begin{bmatrix} 0 & s_1 \\ s_0 & 0 \end{bmatrix}$  are module monomorphisms over complex integers embedding  $\mathbb{Z}[i]^2$  in  $A_G$ .

**Proposition 3.4.**  $A_S^{(\beta)} = \text{kernel}(\zeta^{(\beta)})$  be  $A_T$  submodules of  $A_G$ . Each of these is generated by the  $\mathbb{Z}$ -base  $\{(E_0 - \beta E_1), (-\beta E_2 + E_3), (E_4 - \beta E_5), (-\beta E_6 + E_7)\}$ .  $A_G / A_S^{(\beta)} \simeq \mathbb{Z}[i]^2$  is isomorphic as  $A_T$ -modules.

In the following I restrict the considerations on  $A_S \stackrel{\text{def}}{=} A_S^{(-1)}$ . The suggestion is that only the classes  $A_G / A_S$  are distinguishable by observation and hence represent states.

Overall we now have for  $\forall A \in A_T, B \in A_G: \zeta^{(\alpha)}(AB) = \Psi(A) \zeta^{(\alpha)}(B)$ . That means all internal operations of  $A_T$  may get exposed to the complex plane as the well known matrix operations on classical states.

Finally the following should be recognized

**Proposition 3.5.** *For any  $\beta \in \{\pm 1; \pm i\}$  the combined mapping  $\zeta^{(\beta)} \circ \Psi^{-1}: \text{Mat}_{\mathbb{Z}[i]}(2 \times 2) \rightarrow \mathbb{Z}[i]^2$  is given by  $M \rightarrow (1+i)M \begin{pmatrix} 1 \\ \beta \end{pmatrix}$ .*

<sup>2</sup> f and g are equivalent, if  $f = \alpha g; \alpha \in \mathbb{Z}[i]$

## 4. TENSOR ALGEBRA ON $A_G$

### 4.1. SIMPLIFICATION

At this point I'm going to introduce *simplification* as a generic principle accompanying observations and then look how that applies to a appropriate tensor algebra.

If you look at the mapping between a particle state to its classical state the *modulo*  $A_S$  operation is essential. You can take this by considering  $A_S$  as some sort of a background noise that an observer will never realize.

*Simplification* means that an observer has to simplify the observed environment before observation. It is quite clear that the type of simplifications will have a fundamental influence on the observation. Even slightly different simplifications probably lead to extremely different observations of the same underlying system.

In the general case the *simplification* says that complexity in any environment  $E$  gets simplified for observation by applying a class structure  $\bar{N}$  on  $E$  and dealing with those classes instead of the environment itself.

This way the observed behavior of any physical systems should emerge by simplification from an underlying more complex pattern. It will be shown that there exist underlying patterns from that any behavior deduced from the standard model will emerge.

**Definition 4.1.** Let  $\Gamma = \otimes_1^n A_G$  and  $\Theta = \otimes_1^n A_T$   $\mathbb{Z}$  tensor products. Then  $\Gamma$  is an  $\Theta$  left module. Let  $N \subset \Gamma$  be a  $\Theta$  submodule then a  $\Theta$ -epimorphism  $F: \Gamma \rightarrow \Gamma/N$  is called a “*Simplification of  $\Gamma$  with respect to  $N$* ”.

### 4.2. TENSOR INTERACTION IN $A_G$

In the following sections the discrete structure of the algebra is not important. With real coefficients in  $A_R = \mathbb{R} \otimes A_T$  I need not distinguish between operators and states. As such the previously defined isomorphism extends to  $\Psi: A_R \rightarrow Mat_{\mathbb{C}}(2 \times 2)$  as an  $\mathbb{C}$  algebra.

**Definition 4.2.** Getting back to the definition of classical states I call  $v_\beta \stackrel{\text{def}}{=} (1+i) \begin{pmatrix} 1 \\ \beta \end{pmatrix}; \beta \in \{\pm 1; \pm i\}$  a perspective. Given an observable  $A$  and a system  $B$  as elements in  $Mat_{\mathbb{C}}(2 \times 2)$  I call the triple  $(A, B, v_\beta)$  an observation such that  $Bv_\beta$  represents the classical state.

By this definition it is quite clear how to define scalar and tensor products. By doing this no contradiction with any statement in standard QM should evolve. Coordinate changes are also not an issue here and keep the model consistent.

On the other hand by having the Hilbert product and spinor product on  $\mathbb{C}^2$  these should emerge by a concept of tensors in  $A_R$ . The mapping  $B \rightarrow Bv_\beta$  has a kernel that maps to  $A_S^{(\beta)} = \text{kernel}(\zeta^{(\beta)})$  as  $A_R$  submodule of  $A_R$ . However the concept of tensors relies on invariance under coordinate transformations. That does not really make sense within the GenI algebra because its elements do not operate on vectors that could be subject to transformations. So I will avoid the term “tensor” and just refer to “cubes”.

In the following considerations I restrict  $\beta = -1$ . We will now call  $A_S \stackrel{\text{def}}{=} A_S^{(1)}$  the space of covariant cubes in  $A_R$ . This makes sense as follows:

With  $A = A^{ijk} = \begin{bmatrix} a_0 & a_1 \\ a_2 & a_3 \end{bmatrix} \begin{bmatrix} a_4 & a_5 \\ a_6 & a_7 \end{bmatrix}_3$  as usual let the according covariant cube be  $A_{ijk} = \begin{bmatrix} a_5 - a_4 & a_4 - a_5 \\ a_6 - a_7 & a_7 - a_6 \end{bmatrix} \begin{bmatrix} a_0 - a_1 & a_1 - a_0 \\ a_3 - a_2 & a_2 - a_3 \end{bmatrix} \in A_S^{(1)}$  and take  $A \wedge B := A_{ijk} B^{ijk}$  as the spinor product of  $A$  and  $B$ .

The map  $A^{ijk} \rightarrow A_{ijk}$  has kernel  $A_S$ . So the interaction of  $A$  with  $B$  works in a way that  $B$  operates on  $\bar{A} \stackrel{\text{def}}{=} A \text{ mod } A_S$  resp.  $A$  operates on  $\bar{B} = B \text{ mod } A_S$ .<sup>4</sup>

<sup>3</sup> Here I use the different representation of indices by the identity  $ijk = 4i+2j+k, i,j,k = 0,1$  as binary numbers

<sup>4</sup>  $A$ 's operation only realizes those features of  $B$  exposed from  $A_S$  and vice versa.  $A$  simplifies its perception of  $B$  by looking at classes of

**Proposition 4.1.** *The defined product is antisymmetric  $A \wedge B = -B \wedge A \forall A, B \in A_R$ . It induces an operation on  $A_R/A_S$  by  $\bar{A} \wedge \bar{B} := A \wedge B$ . This is a consistent definition because for  $\forall A \in A_R, B' \in A_S: A \wedge B' = 0$  and hence  $(A + A') \wedge (B + B') = A \wedge B \forall A', B' \in A_S$ . You find for the map  $\zeta: A_R \rightarrow \mathbf{C}^2$  that the classical spinor product is*

$$\zeta(A)^i \zeta(B)_i = A_{ijk} B^{ijk} + i[(i A_{ijk}) B^{ijk}] \text{ where consequently } iA_{ijk} = \begin{bmatrix} a_7 - a_6 & a_6 - a_7 \\ a_5 - a_4 & a_4 - a_5 \end{bmatrix} \begin{bmatrix} a_2 - a_3 & a_3 - a_2 \\ a_0 - a_1 & a_1 - a_0 \end{bmatrix}.$$

Next look for a proper symmetric scalar product compliant with the standard model and look if it makes sense in  $A_R$ .

**Definition 4.3.** With the transformation  $A_{ijk}^* = \begin{bmatrix} a_0 - a_1 & a_1 - a_0 \\ a_2 - a_3 & a_3 - a_2 \end{bmatrix} \begin{bmatrix} a_4 - a_5 & a_5 - a_4 \\ a_6 - a_7 & a_7 - a_6 \end{bmatrix}$  I call  $\llbracket A|B \rrbracket := A_{ijk}^* B^{ijk}$  the symmetric scalar product and  $\llbracket B \rrbracket = \sqrt{\llbracket B|B \rrbracket}$  the (pseudo-)norm of B.

Again  $\forall A \in A_R, B \in A_S: A_{ijk}^* B^{ijk} = 0$  makes that portable to  $A_R/A_S$ . The product is symmetric. Further I have the standard Hilbert product recovered as  $\zeta(A)^i \zeta(B)_i = A_{ijk}^* B^{ijk} + i[(i A_{ijk}^*) B^{ijk}]$ .

## 5. MEASUREMENT DYNAMICS

Until now there is nothing really new at the bottom line. The presented model is speculative, incorporates additional complexity and seems to provide nothing beyond the actual understanding of quantum physics, reality (as described by the GR) and consciousness. Let me start with some dynamics that at some point need the GenI model to keep definitions simple and straightforward.

Wave reduction of a system is a instantaneous process in the QM standard model at the moment a measurement gets executed. Decoherence models introduce dynamics induced by an environment. I'm going to introduce a alternative model for a state reduction process here that will get justified later.

I start with an observation  $(A, B, \nu)$  where a system  $A \in \text{Mat}_{\mathbb{Z}[i]}$  operates on a system  $B \in \text{Mat}_{\mathbb{Z}[i]}$  via a perspective  $\nu \in \mathbb{Z}[i]^2$ . I view  $A$  and  $B$  as being collections of simple base elements – say the matrix representations of the 16 Pauli group members. For each such member  $P$  also  $-P$  is a member. The operation of  $A = \{A_j\}$  on  $B\nu$  is the matrix operation  $\sum_j A_j$  and each matrix can be constructed in this (non-unique) way.  $B$  is also considered as a set  $B = \{B_k\}$  and its vector representation is recovered by  $B\nu = \sum_k B_k \nu$ . Now the model is that the measurement by  $A$  causes some dynamics within  $B$ . These result in creation and deletion of some of its members in a well defined but random way. This makes sense because I deal with a integer model.

Back to  $B$ 's representation as a matrix such a process will result ultimately in changing matrix elements or integer values of cube nodes in the GenI model.

### 5.1. A STOCHASTIC MODEL

In this section I'm going to show the door that is not there in the standard model. At this point I will draft a stochastic model for state reduction.

Let  $A, B \in A_R$  real GenI states. In this case both have also operational features and let  $A$  be hermitian.  $A$  operates on  $B$  and  $B$  undergoes a wave reduction according to the eigenvectors of  $A$ . This is the standard model view and I call it the  $A$ -view. Given  $A$  and  $B$  are basically equivalent there should be a  $B$ -view.

And here is a proposal for the  $B$ -view:

The system  $B$  should not know anything about  $A$ 's internal structure. Only a weak feedback by scalar values on  $B$ 's behavior is acceptable. By means of excitations and a scalar field  $A$  governs the rules of a stochastic process that leads to a new final state of  $B$ . The final result has to fully comply with the predictions of the QM standard model.

Several suggestions accompanied by a lot of simulations finally come out with the following proposal. It basically implements a random walk stochastic process starting with  $B$ 's actual state:

Let  $B$  receive some excitation by  $A$  according to the angle between the straight lines along  $AB$  and  $B$  respectively. The angle  $\phi_A(B) \in [0; \pi/2]$  gets measured by  $\cos(\phi_A(B)) = \frac{\|AB\| \|B\|}{\|AB\| \|B\|}$  using the symmetric scalar product. With respect to “simplification” I'd say  $A$  recognizes  $B$  as  $B \text{ mod } (A_S)$ .

The excitation defined as  $e_A(B) := \|B\| (\sin \phi)^2$  keeps  $B$ 's state moving randomly (uniformly distributed) by  $|\Delta B| \leq e_A(B)$ .

Now in a random walk manner  $B$  decides to move on any straight line along any direction. Again  $A$  feeds back on the variation  $B \pm \Delta B$  by the angles  $\phi_A(B \pm \Delta B)$ .  $B$  selects the actual variation randomly with probability  $p(\pm \Delta B) = \frac{\phi_A(B \mp \Delta B)}{\phi_A(B + \Delta B) + \phi_A(B - \Delta B)}$ . So the lower value of  $\phi_A(B)$  get selected with higher probability.

The process can be simulated and indeed shows the expected behavior. An eigenvector to  $A$  is reached with a probability according to  $|\lambda_i|^2$  in  $B = \lambda_0 b_0 + \lambda_1 b_1$ . The graphic below shows the actual probability values as the

final result of the process versus the theoretical values  $\frac{x^2}{x^2+1}$  for  $x = \frac{|\lambda_0|}{|\lambda_1|} \in [0; 1]$ .

The results shown here got generated by performing 1000 runs for each value  $x$  in a selected set and averaging on  $x$ . There is a very good convergence to eigenvectors. No test really gets lost without reaching such an attractor.<sup>5</sup>

Now look what is happening:

Within the  $B$ -view you observe random movement of states with a tendency towards attractors. A model for this would probably introduce forces as a source and has to introduce some concept of time and space to model the movement itself. However these are basic features of the  $B$ -view without any meaning in the  $A$ -view. The reality for  $B$  is this  $B$ -view including forces, space and time. The rules of that process are fixed with respect to a given set of attractors. However every decision by  $B$  to step is basically free. The process is not predictable at any point except when an attractor has been reached.

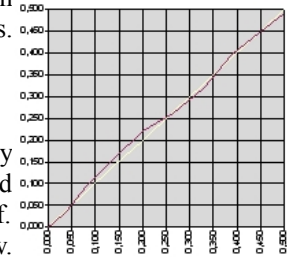


FIGURE 4: STATE REDUCTION

The reality for  $A$  is the quantum mechanical  $A$ -view with wave reduction happening instantaneous. That makes sense because the time variable in  $B$  has no meaning for  $A$ .

If you look at the symmetry group of  $A$  you find them preserved in the symmetries of the attractors in the  $B$ -view. All the orthonormal symmetries of  $A$  don't even change the dynamics of the  $B$ -view. Thus the symmetries of the governing operator  $A$  are perceivable in  $B$ . That is not true vice versa: The  $A$ -view does not allow to see the symmetries of  $B$  directly.

So what if the universe as we know it (and is modeled by GR) is a  $B$ -view? You could argue that you now must introduce  $A$  as something external that would contradict the notion of "universe". But that is not necessarily true.  $A$  and  $B$  may be one entity. Then the  $A$ -view and the  $B$ -view are dual to each other. The  $A$ -view is simply encoded by the  $B$ -view and vice versa. Especially gravity may then be a force observed from the  $B$ -view that does not have any quantum representation inside the system. The same should then be true for mass. If you still look for the graviton you will not find it within the  $B$ -View. You have to switch to the  $A$ -view, i.e. the graviton may be as big as some billions of light years in diameter. However space would not have a meaning in that context.

## 5.2. So WHAT?

The presented model is to a large extent motivated by the assumption of discreteness of physics[6] and that a more complete QM model should not distinguish between observables and observed systems. Doing this the intermediate results look weird and complicated.

However the standard model emerges in a natural way. At the bottom line both are actually very close and this is a good thing. The obvious question is: What should it be good for? Is it adding just an additional layer of hidden complexity? What are the improvements?

A major achievement of this model lies in allowing an alternate view on state reduction as a symmetric interaction among states. It allows to investigate further in wave reduction as a stochastic process with well defined rules. To construct a relatively straightforward random walk process you indeed need that additional degrees of freedom for a representation of states.

The GenI model may open a door to an improved understanding of the roles of QM and GR in a complete picture of reality. Finally I'm not sure if this all is of value to a community. At least it is of value for me to continue with considering intelligent decision taking as a basic physical feature and looking what that means to a concept of reality.

Some work ahead will focus on exactly defining the drafted process, its features and how a curved spacetime evolves within the  $B$ -view.

<sup>5</sup> A simulation is available as a Java applet. Use button "GenI Start" on <http://bzus.de/>

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NELKENWEG 13, 52385 NIDEGGEN, GERMANY  
*E-mail address:* bzus@genreith.com